Optimal trading with a battery: An optimization model for offering flexibility on the day-ahead, intraday and reserve markets

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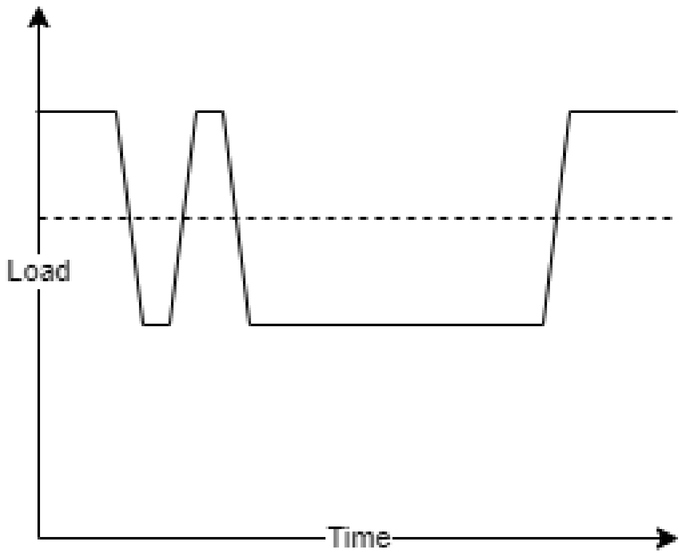
Using an Aluminium Electrolyser like a Battery

- Aluminium electrolysis is an energy intensive process
- Requires continuous energy input
- Temperature must be between 940-980°C

Figure: Aluminium Production ©Trimet SE

Using an Aluminium Electrolyser like a Battery

- Variation of baseload is possible
- Operation at *baseload* $\pm$ *x MW*
- Assuming the baseload has been purchased in advance
  - We **sell** when we use less than the baseload
  - We **buy** when we use more than the baseload
- Process is precise (low ramp up times) and has a large storage capability (Can run at max or min load for over two days)



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Markets for Flexibility

We consider three German electricity markets to use the flexibility:

- Day-Ahead Market
- Intraday Market
- Market for Frequency Restoration Reserve with automatic activation (aFRR)

→ We consider them individually and then we try to optimize the combined use

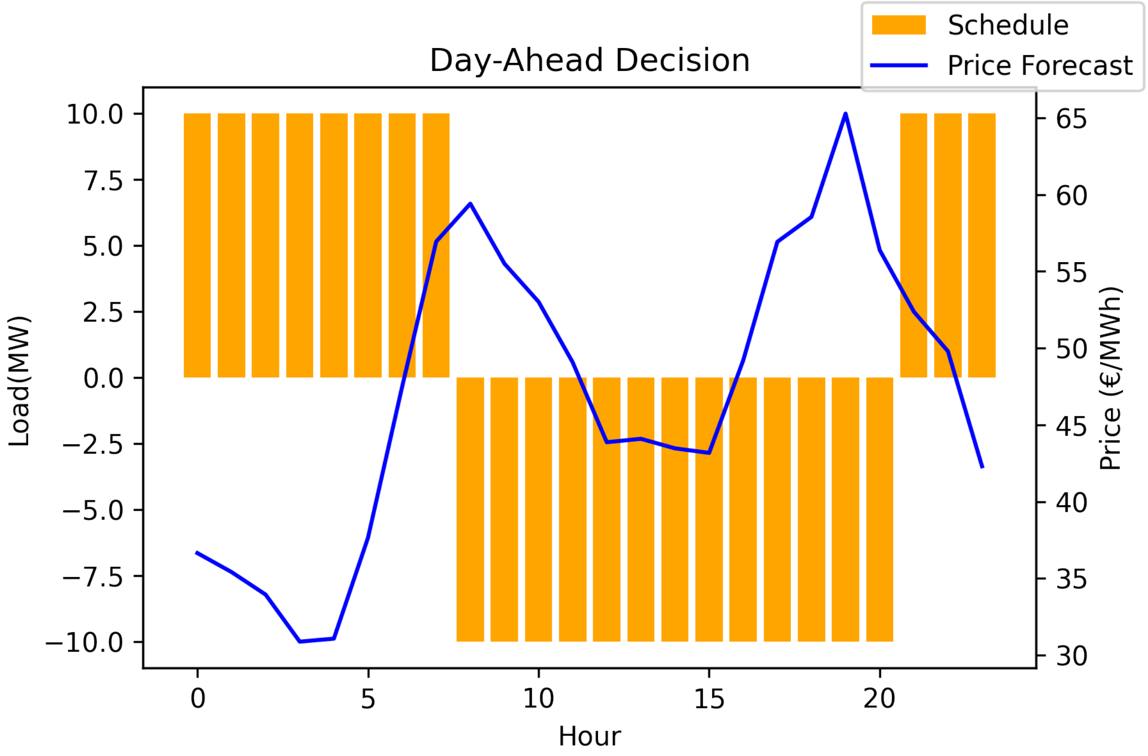
The Day-Ahead Market

Strategy for the Day-Ahead market auction

- Forecast the hourly price
- Buy hours below median cost
- Sell hours above median cost
- Profit forecast:

$$f^{\text{DA}} := \sum_{t=0}^{T} p_t \cdot m_t^{\text{DA}} - \textit{Technical Costs}$$

- Observe battery constraints
- Battery neutral over 7 days



The Intraday Market

Make use of continuous trading:

- Quarter-hourly products can be traded from 16:00 on the day before until 5 minutes to delivery
- We use the price difference between products
- Price swings during the day allow to switch positions several times before the final schedule is set
- An algorithm traverses historical orderbook data and finds concurrent bids where a position switch is profitable

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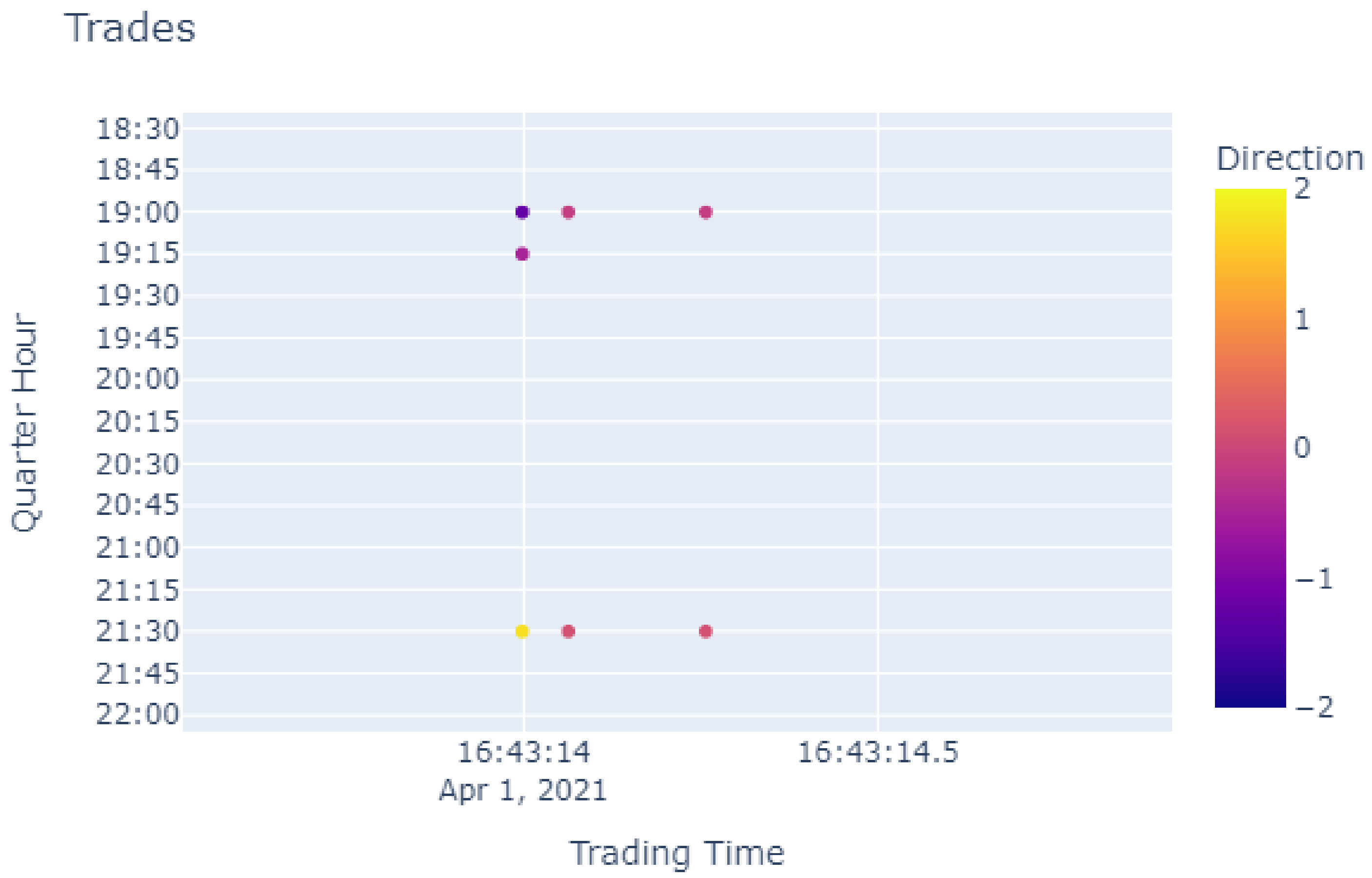
The Intraday Market: Trading Opportunities



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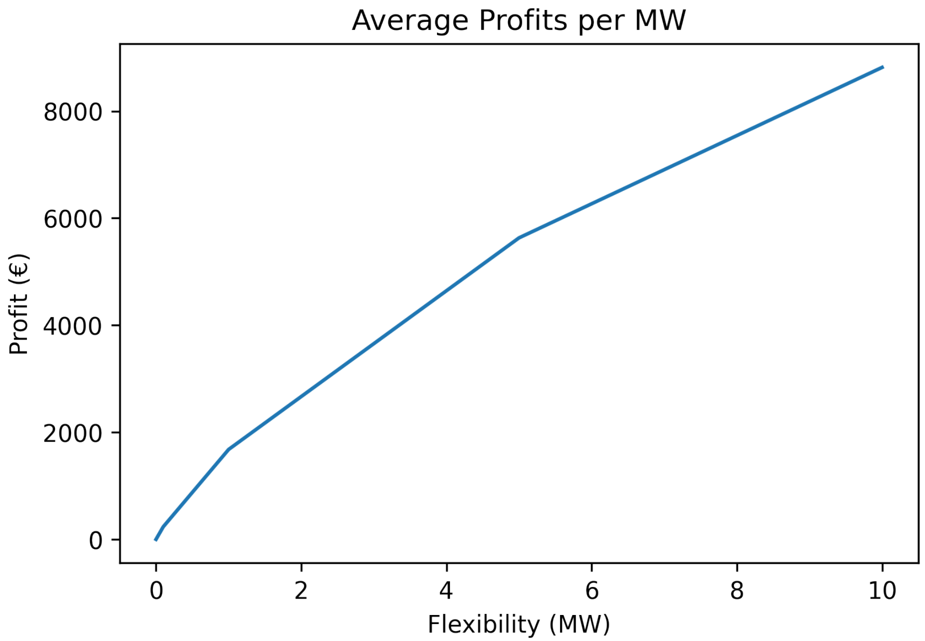
The Intraday Market: Diminishing Returns to Scale

- Run this algorithm with different amounts of flexibility
- Compute average daily profits for the result
- To compute a profit forecast, we forecast a result with 1 MW and scale by this function



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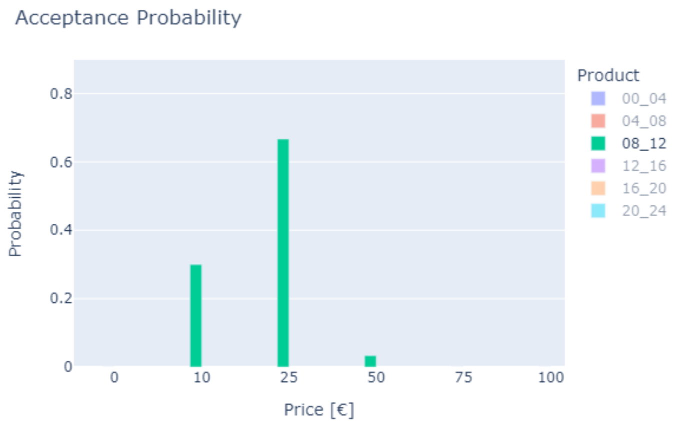
The aFRR Market Auction

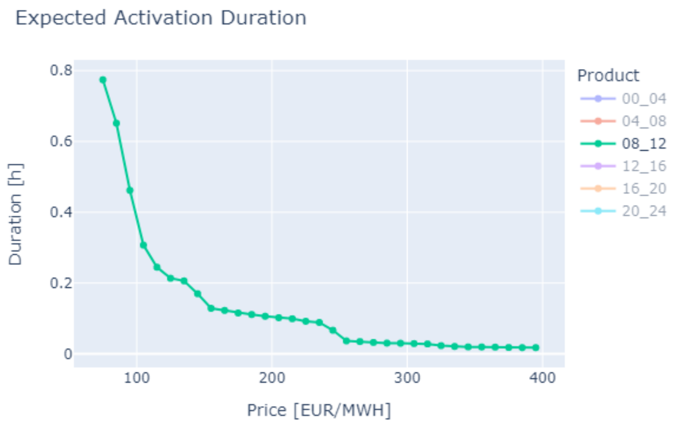
- Place price quantity bids (p, m) in **capacity** and **energy** market on 4 hour products
- Choose optimal **quantity** for given set of price levels
- Maximize expected profit
- Capacity market profit:

$$f^{Cap} := \sum_{i=1}^{N} q_i^{Cap}(p_i) \left(\sum_{j=1}^{i} m_j^{Cap} \cdot p_j^{Cap} \right)$$

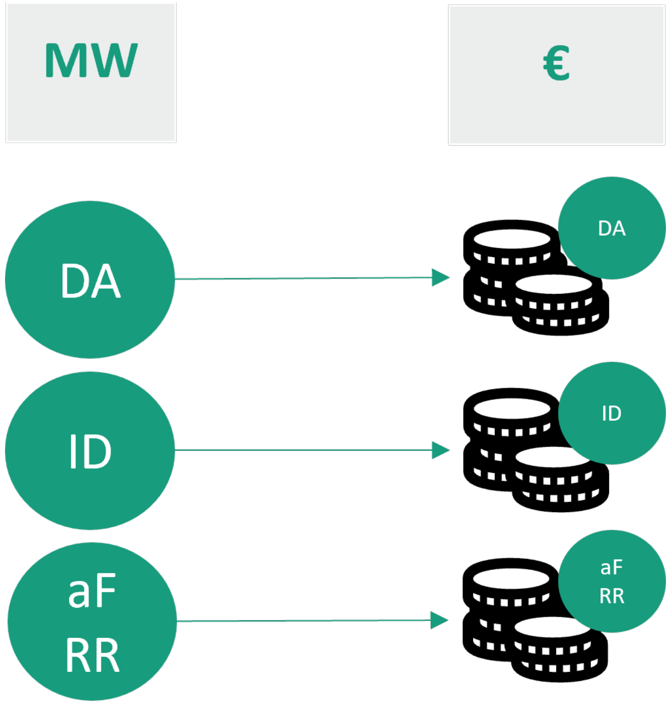
- Energy market profit:

$$f^{En} := \sum_{i=1}^{N} p_i^{En} \cdot m_i^{En} \cdot \alpha_i(p_i)$$





Bringing it all together: Cross-Market Optimization



Two ideas:

- In the Intraday market profits do not scale linearly with the amount of flexibility. $\Rightarrow$ Should we split the flexibility between markets?
- Can we identify days where we should use one market over the others?

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Formulate the Optimization Problem

Inputs:

- Profit forecast for each market:
  - $f^{\mathrm{aFRR}}, f^{\mathrm{ID}}, f^{\mathrm{DA}}$
  - Piecewise linear in m, amount of allocated flexibility
- Maximum flexibility of the process: M
- Limits and state of charge for the battery: B_{min}, B_{max}, B_0
- A planning horizon T (e.g. one week $T = 7 * 24$)
- Number of price levels for aFRR N

Decision Variables:

- Allocation per market:
  - $m^{\mathrm{aFRR}}, m^{\mathrm{ID}}, m^{\mathrm{DA}}$
- Allocation within the market
  - $m_t^{\mathrm{DA}}, \; t = 1, \dots, T$
  - $m_i^{Cap,k}, \; k = 1, \dots, 6, i = 1, \dots, N$
  - $m_i^{En,k}, \; k = 1, \dots, 6, i = 1, \dots, N$

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Formulate the Optimization Problem

$$\textit{Expected Profit}: \quad \max_{m^{\mathrm{aFRR}}, m^{\mathrm{ID}}, m^{\mathrm{DA}}} \; f^{\mathrm{aFRR}}(m^{\mathrm{aFRR}}) + f^{\mathrm{ID}}(m^{\mathrm{ID}}) + f^{\mathrm{DA}}(m^{\mathrm{DA}})$$

$$\textit{Power Constraint}: \quad m^{\mathrm{DA}} + m^{\mathrm{ID}} + m^{\mathrm{aFRR}} \leq M$$

$$\textit{DA Market Opt}: \quad \max_{m_t^{\mathrm{DA}}} \sum_{t=0}^{T} p_t \cdot m_t^{\mathrm{DA}} - TC$$

$$\textit{aFRR Market Opt}: \quad \max_{m_i^{Cap,k}, m_i^{En,k}} \sum_{i=1}^{N} q_i^{Cap}(p_i) \left(\sum_{j=1}^{i} m_j^{Cap} \cdot p_j^{Cap} \right) + \sum_{i=1}^{N} p_i^{En} \cdot m_i^{En} \cdot \alpha_i(p_i)$$

$$m^{\mathrm{DA}} \geq max(m_t^{\mathrm{DA}})$$

$$m^{\mathrm{aFRR}} \geq max(m_j^{Cap,k})$$

$$\textit{Battery Constraint}: \quad B_{\min} + 12m^{ID} \leq B_0 + \sum_{t=0}^{k} \left(m_t^{\mathrm{DA}} + m_{\lfloor t/4 \rfloor}^{Cap} \right) \leq B_{\max} - 12m^{ID} \qquad \forall k = 0, \ldots, T$$

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Formulate the Optimization Problem

$$\textit{Expected Profit}: \quad \max_{m^{aFRR}, m^{ID}, m^{DA}} f^{aFRR}(m^{aFRR}) + f^{ID}(m^{ID}) + f^{DA}(m^{DA})$$

$$\textit{Power Constraint}: \quad m^{DA} + m^{ID} + m^{aFRR} \leq M$$

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$$m^{DA} \geq max(m_t^{DA})$$

$$m^{aFRR} \geq max(m_j^{Cap,k})$$

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$$m^{DA} \geq max(m_t^{DA})$$

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Formulate the Optimization Problem

$$
\begin{aligned}
&\textit{Expected Profit}: && \max_{m^{\mathrm{aFRR}}, m^{\mathrm{ID}}, m^{\mathrm{DA}}} f^{\mathrm{aFRR}}(m^{\mathrm{aFRR}}) + f^{\mathrm{ID}}(m^{\mathrm{ID}}) + f^{\mathrm{DA}}(m^{\mathrm{DA}}) \\
&\textit{Power Constraint}: && m^{\mathrm{DA}} + m^{\mathrm{ID}} + m^{\mathrm{aFRR}} \leq M \\
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\end{aligned}
$$

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Formulate the Optimization Problem

Expected Profit :
$$\max_{m^{\mathrm{aFRR}},m^{\mathrm{ID}},m^{\mathrm{DA}}} f^{\mathrm{aFRR}}(m^{\mathrm{aFRR}}) + f^{\mathrm{ID}}(m^{\mathrm{ID}}) + f^{\mathrm{DA}}(m^{\mathrm{DA}}) - OC(EI)$$

Energy Imbalance :
$$EI := \sum_{t=0}^{24} m_t^{\mathrm{DA}} + \sum_{k=1}^{6} \sum_{i=1}^{N} \cdot m_i^{En,k} \cdot \alpha_i^k$$

OC is the opportunity cost of an energy imbalance at the end of the day

$$OC(EI) = f^{\mathrm{ID}}(EI)$$

Formulate the Optimization Problem

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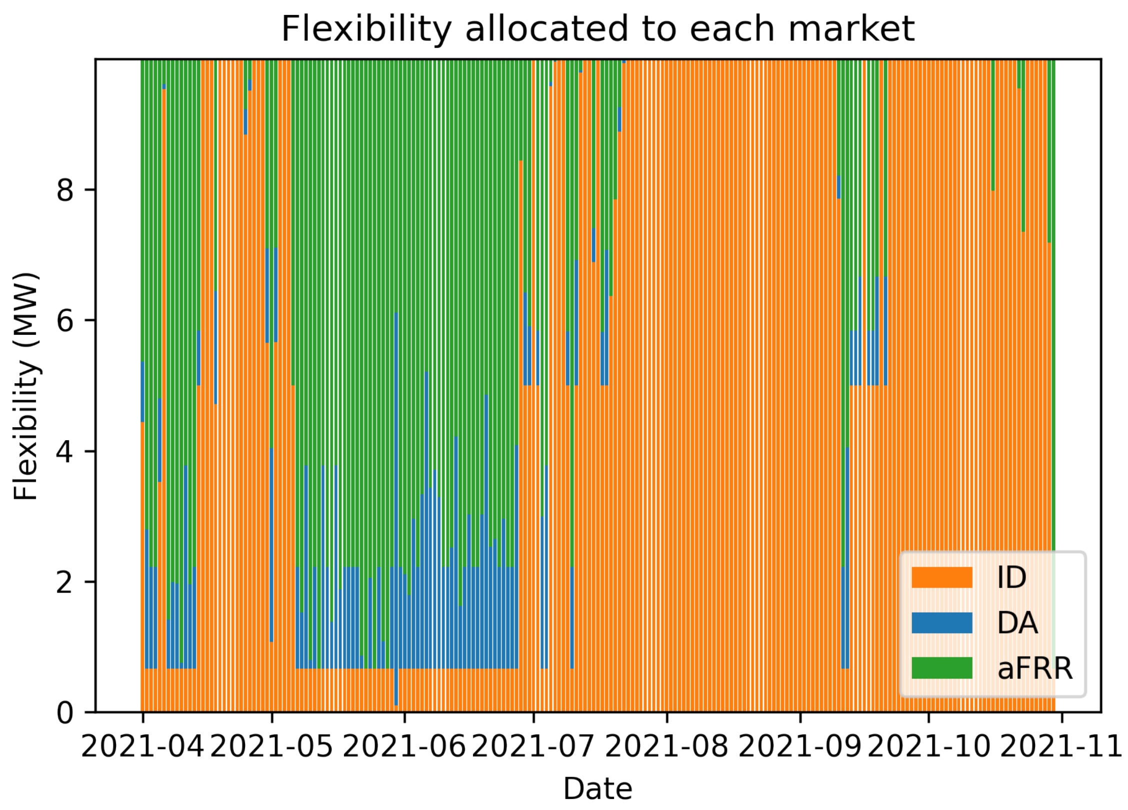
Backtesting the Strategy

Evaluation period: **01.04.2021-30.10.21**
Evaluate on a rolling basis:

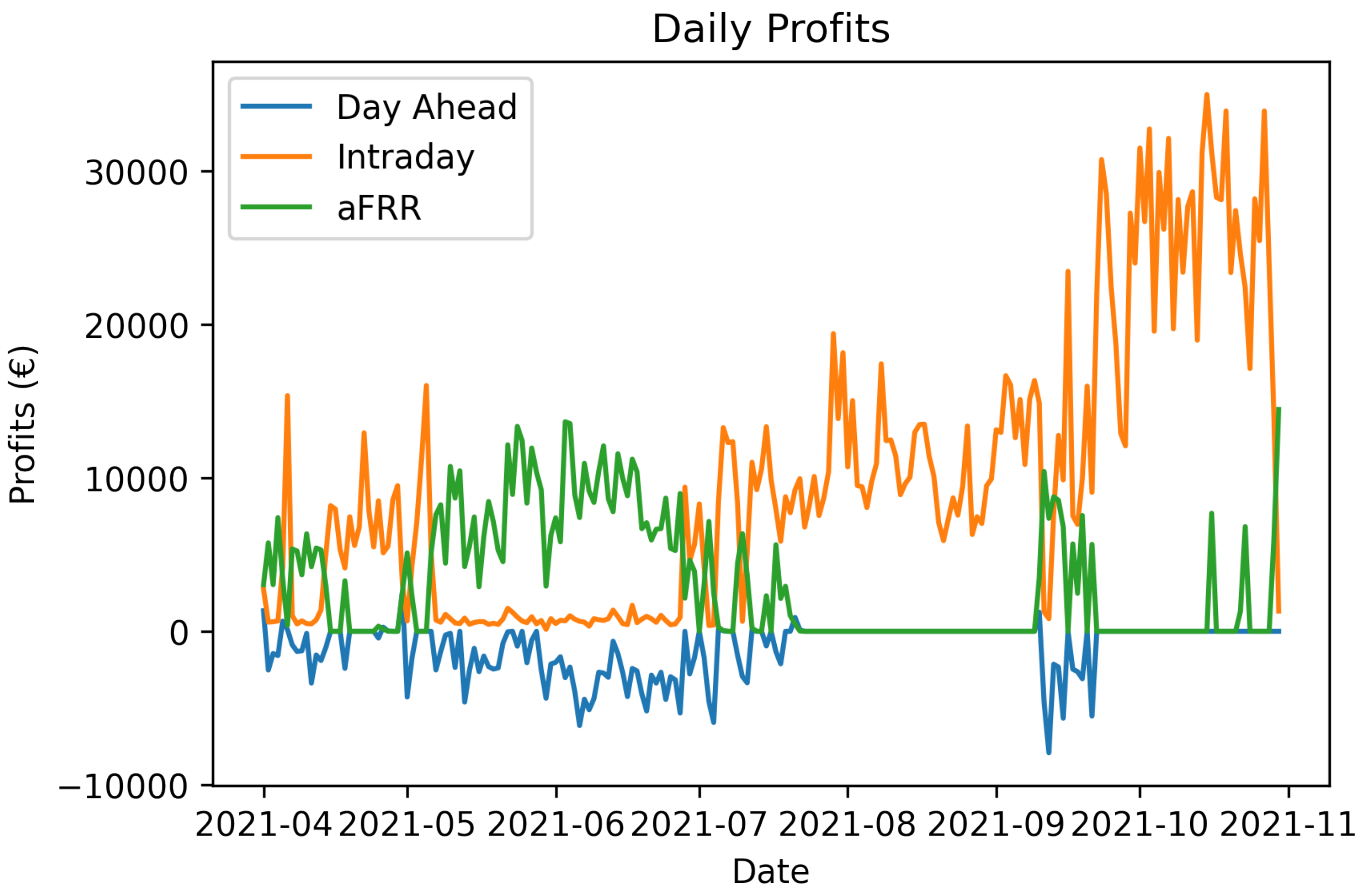
1. Solve optimization problem
2. Adjust battery level with aFRR activation data

Average Flexibility per Market

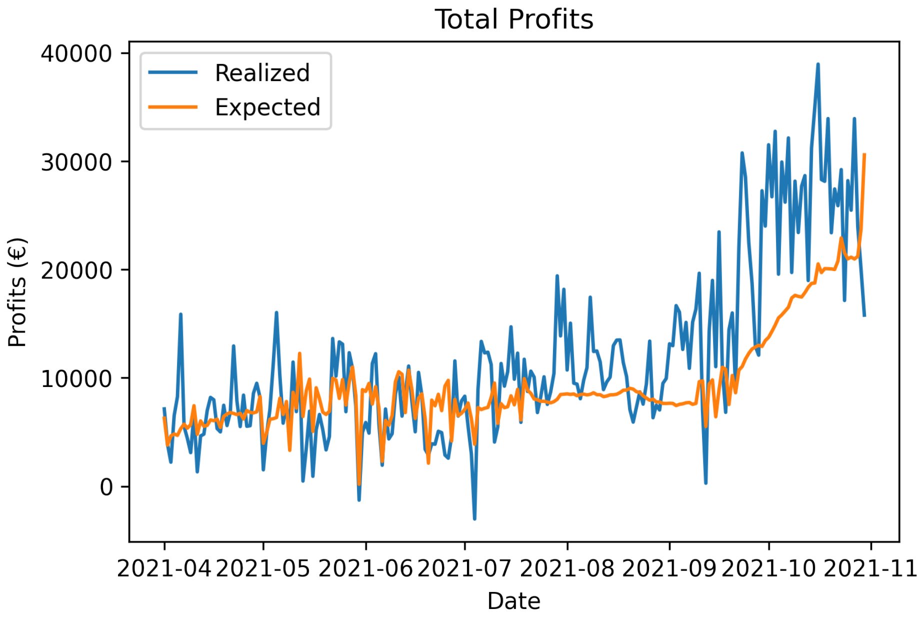
Day-Ahead	Intraday	aFRR
0.7 MW	6.9 MW	2.4 MW

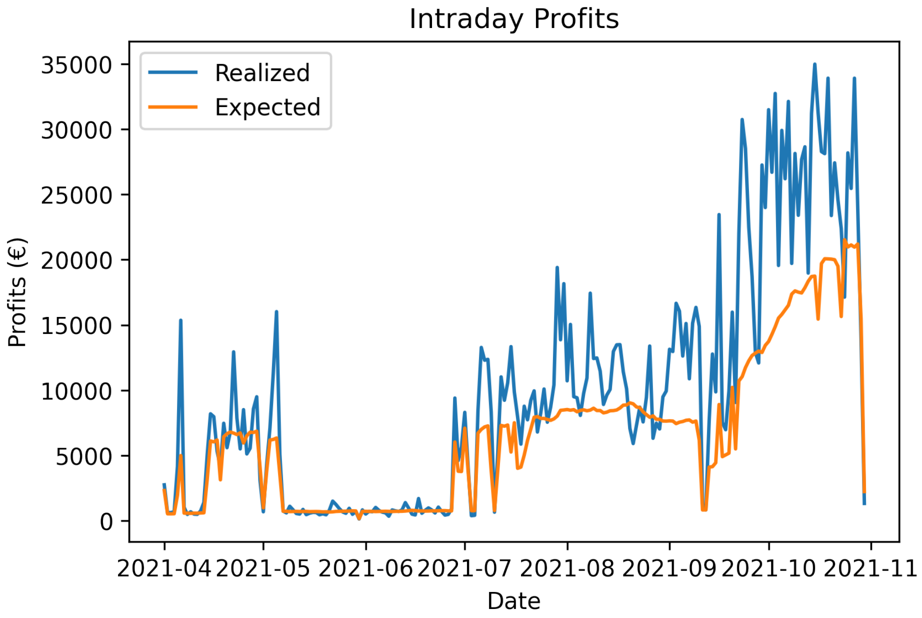


Analysing Profits



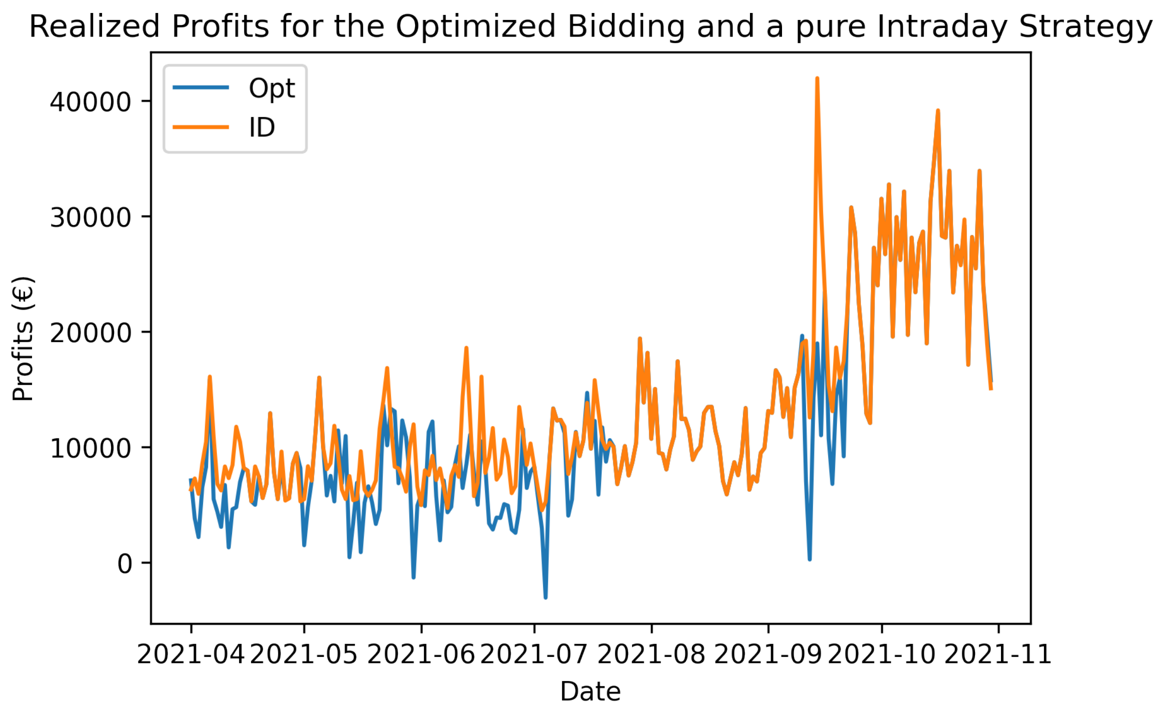
Forecast vs Realized





Comparison with an Intraday-Only Strategy

Strategy	Real Profits	Exp Profits
Opt	2.53e+06	2.00e+06
ID Only	2.82e+06	1.93e+06



Summary

- The intraday-market yields the highest profit opportunities in our setting
- Certain days can be identified where using several markets is optimal
- Quality of forecasts is crucial for the optimization decision
- Value of flexibility goes up from 04/2021-10/2021, clear connection with rising prices

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Forecast vs Realized (additional)