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Modeling Volatility and Dependence of European Carbon and Energy Prices

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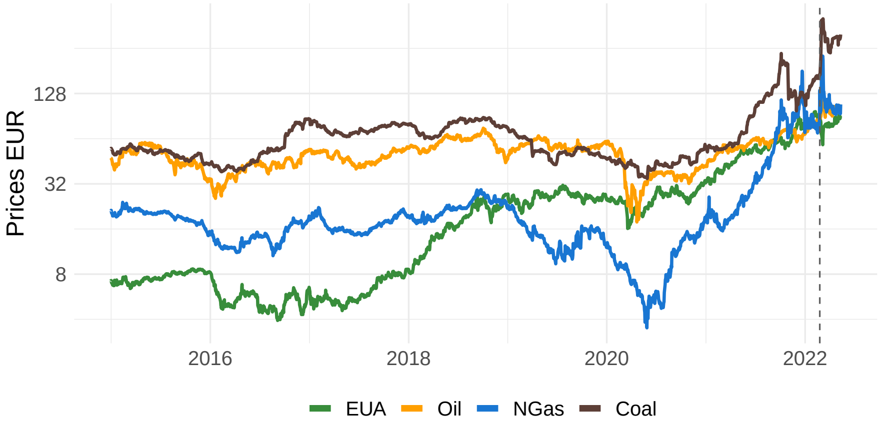
Contents

Motivation & Data

Motivation & Data

- Understanding European Allowances (EUA) dynamics is important for several fields:
  - Portfolio & Risk Management,
  - Sustainability Planing,
  - Political decisions,
  - ...
- How can the dynamics be characterized?
- On their own, EUA prices are 'just' a random walk.
- The first differences are stationary and heteroskedastic.
- Autoregressive Volatility modeling (ARCH, GARCH, ...) is sensible!

- EUA prices are obviously connected to the energy market Chevallier (2019),
- But what is the exact form of the connection?
- We consider multivariate autoregressive (VAR) and cointegrating (VECM) relations for mean modeling.
- We further consider copulas for contemporaneous stochastic dependence structure modeling.
- In total our model is a VECM-Copula-GARCH model.
- The model generalizes the copula-GARCH model, Aloui et al. (2013), Jondeau & Rockinger (2006), Hu (2006).


Prices EUR
128
32
8
2016
2018
2020
2022
EUA
Oil
NGas
Coal

Methods

Statistical Methods

- Let $\mathbf{X}_t = (X_{1,t}, \ldots, X_{d,t})$ be a d-variate time series,
- Let the conditional joint distribution of $\mathbf{X}_t|\mathcal{F}_{t-1}$ be denoted by $F_{\mathbf{X}_t|\mathcal{F}_{t-1}}$.
- The conditional marginal distributions of $X_{i,t}|\mathcal{F}_{t-1}$ are denoted by $F_{X_{i,t}|\mathcal{F}_{t-1}}$.
- Then by Sklars Theorem, Sklar (1959),

$$\begin{aligned} F_{\mathbf{X}_t|\mathcal{F}_{t-1}}(\mathbf{a}|\mathcal{F}_{t-1}) = C[&F_{X_{1,t}|\mathcal{F}_{t-1}}(a_1;\mu_{1,t},\sigma_{1,t},\vartheta_1),\ldots, \\ &F_{X_{d,t}|\mathcal{F}_{t-1}}(a_d;\mu_{d,t},\sigma_{d,t},\vartheta_d);\Xi_t,\Theta] \end{aligned} \tag{1}$$

- The model is specified by its marginal specifications $F_{X_{i,t}|\mathcal{F}_{t-1}}(a_i; \mu_{i,t}, \sigma_{i,t}, \vartheta_i)$ and dependence structure C.
- The following is comprised of descriptions of
  - Marginal distribution $F_{X_{i,t}|\mathcal{F}_{t-1}}$,
  - mean modeling $\mu_{i,t}$,
  - volatility modeling $\sigma_{i,t}$,
  - dependence structure C and
  - time varying dependence parameters Ξ_t.

Marginal Specification

- We choose the generalized t-distribution, Theodossiou (1998), $t_{\mu,\sigma,\lambda,\nu}$ as marginal distribution,
- Reparametrize such that $\mu \leftrightarrow$ mean and $\sigma^2 \leftrightarrow$ variance.
- λ and ν can be interpreted as skewness and heavy-tailedness.
- Mean and Variance are modeled time varying
- Skewness and heavy-tailedness are assumed to be constant.

Mean modeling

- The means are modeled with a vector error correction model (VECM),

$$\Delta\boldsymbol{\mu}_t := \Delta\mathbf{x_t} = \Pi x_{t-1} + \Gamma\Delta x_{t-1}. \tag{2}$$

- where $\Pi = \alpha\beta$ with $\alpha, \beta \in \mathbb{R}^{d\times r}$ and r is the cointegration rank.
- β comprises the cointegrating relations, hence βx_{t-1} is stationary.
- α determines the speed of adjustment to the equilibrium.
- The model accounts for autoregressive influences with $\Gamma \in \mathbb{R}^{d\times d}$ as well as long term relations with Π.

Volatility Modeling

- The volatility is modeled in a univariate manner,
- Each time series volatility is modeled by a leverage GARCH process,

$$\sigma_{i,t}^2 = \omega_i + \alpha_i^+ (\epsilon_{t-1}^+)^2 + \alpha_i^- (\epsilon_{t-1}^-)^2. \tag{3}$$

- Negative shocks can have stronger/weaker influences on the volatility.
- For $\alpha_i^+ = \alpha_i^-$ the usual GARCH is recovered.

Dependence Modeling

- In this work we choose the t-copula Demarta & McNeil (2005), as dependence structure, $C = C^t$.
- The t-copula allows for linear and heavy-tailed dependence,
- It is constructed using the multivariate t-distribution,

$$C^t[u_1, \dots, u_d] = t_{\Sigma,\kappa}(t_\kappa^{-1}(u_1), \dots, t_\kappa^{-1}(u_d)) \tag{4}$$

- The lower the degree of freedom κ, the heavier the tails, increasing the coincidence of extreme events.

Time Varying Dependence

- The matrix governing the linear dependence is allowed to be time-varying, $\Sigma = \Sigma_t$,

$$\Sigma_t = \Lambda(\rho_t) \tag{5}$$

$$\rho_{ij,t} = \eta_{0,ij} + \eta_{1,ij}\rho_{ij,t-1} + \eta_{2,ij} z_{i,t-1} z_{j,t-1} \tag{6}$$

- where Λ is a suitable link function assuring that Σ_t is positive semi-definite,
- $z_{i,t-1}$ is the standardized residual from time series i, $i \in \{1, \dots, d\}$ at time $t-1$,

$$z_{i,t-1} = \frac{x_{i,t-1} - \mu_{i,t-1}}{\sigma_{i,t-1}}. \tag{7}$$

- Several models are nested by the VECM-Copula-GARCH model:
- $\Gamma = 0, \Pi = 0 \rightarrow \text{RW}^{\sigma_t,\rho_t}_{\text{ncp,lev}}$,
- $\Gamma = 0, \Pi = 0, \boldsymbol{\alpha}^+ = 0, \boldsymbol{\alpha}^- = 0, \boldsymbol{\beta} = 0 \rightarrow \text{RW}^{\sigma,\rho_t}_{\text{ncp,lev}}$,
- $\boldsymbol{\alpha}^+ = 0, \boldsymbol{\alpha}^- = 0, \boldsymbol{\beta} = 0, \nu \rightarrow \infty, \boldsymbol{\eta}_1 = 0, \boldsymbol{\eta}_2 = 0,$
  $\boldsymbol{\kappa} \rightarrow \infty, \textbf{ncp} = 1, \rightarrow \text{VECM}^{\sigma,\rho}$

⋮

Estimation

- The model (and all nested models) are estimated in a one-step procedure with maximum likelihood estimation.
- Hence there is no transmission of estimation errors.
- Also, features of the data are attributed to the correct model component.

Forecasting Study

- We compare the model and nested models forecasting performance,
- Further we include an exponential smoothing (ETS) model.
- 1-30 day-ahead forecasts are examined in a rolling window forecasting study.
- The forecasts are approximated by Monte-Carlo simulations.
- The window size is 1000,
- Forecasts are conducted from 250 observations.

- Point forecasts are evaluated by the RMSE,
- univariate probabilistic forecasts are evaluated by the CRPS,
- Temporal as well as cross-sectional multivariate forecasts are evaluated by the Energy Score

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{x}_i - x_i)^2}, \tag{8}$$

$$\text{CRPS} = \int dt(\hat{F}(t) - F(t))^2, \tag{9}$$

$$\text{ES} = \frac{1}{2}E_{\hat{F}}||\mathbf{X} - \mathbf{X}'|| - E_{\hat{F}}||\mathbf{X} - \mathbf{x}||. \tag{10}$$

Results

Model	ES^{All}_{1-30}	ES^{EUA}_{1-30}	ES^{Oil}_{1-30}	ES^{NGas}_{1-30}	ES^{Coal}_{1-30}	ES^{All}_{1}	ES^{All}_{5}	ES^{All}_{30}
$RW^{\sigma,\rho}$	1.11	0.37	0.39	0.71	0.47	0.04	0.11	0.28
$RW^{\sigma_t,\rho}_{lev,ncp}$	**0.76**	0.59	3.83	**0.43**	-0.24	2.12	1.94	0.71
$RW^{\sigma_t,\rho_t}_{lev,ncp}$	0.71	0.88	**4.15**	0.20	-0.26	**2.27**	**1.96**	**0.80**
$RW^{\sigma_t,\rho}_{ncp}$	0.25	0.38	3.32	-0.20	-0.23	2.24	1.85	0.02
$RW^{\sigma_t,\rho_t}_{ncp}$	0.22	0.47	3.22	-0.24	-0.10	2.14	1.86	0.10
$VECM^{r0,\sigma,\rho_t}$	0.07	0.52	-0.37	0.16	**0.54**	-0.63	-0.48	0.11
$VECM^{r1,\sigma,\rho_t}_{ncp}$	-4.20	-3.67	-2.36	-4.92	-0.07	-1.28	-1.40	-6.16
$VECM^{r2,\sigma,\rho_t}_{ncp}$	-4.63	0.67	-1.91	-7.06	-0.65	-0.61	-1.32	-6.38
$VECM^{r3,\sigma_t,\rho}$	-6.35	-2.32	-1.53	-9.51	-0.91	0.38	-0.15	-9.95
$VECM^{r4,\sigma,\rho_t}$	-9.61	-4.88	-3.83	-14.67	-2.80	-1.00	-2.42	-14.60
ETS^{σ}	-5.21	**2.88**	-0.04	-2.88	-0.40	-3.74	-5.50	-9.26

Coloring w.r.t. test statistic: <-5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | >5

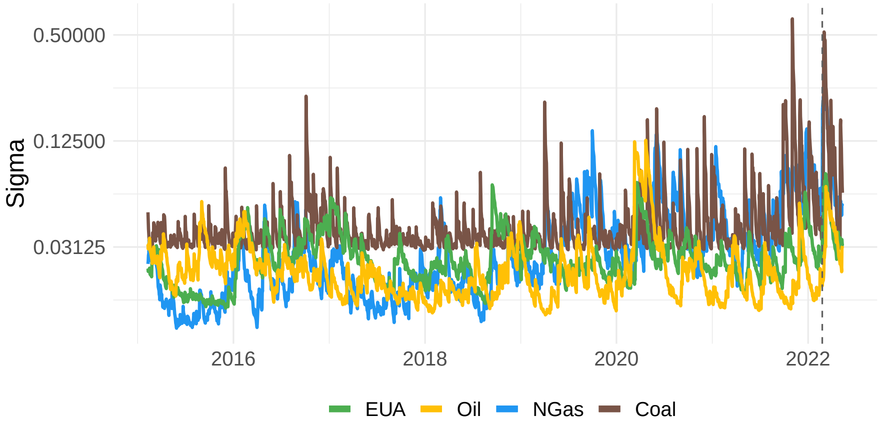
CRPS	EUA			NGas			Oil			Coal		
Model	H1	H5	H30	H1	H5	H30	H1	H5	H30	H1	H5	H30
$RW^{\sigma,\rho}$	0.01	0.04	0.08	0.01	0.03	0.09	0.03	0.07	0.17	0.01	0.03	0.12
$RW^{\sigma_t,\rho}_{lev,ncp}$	0.67	0.14	1.33	3.62	5.74	4.48	3.09	**2.42**	-0.04	1.04	**1.06**	-0.75
$RW^{\sigma_t,\rho_t}_{lev,ncp}$	1.17	0.58	1.67	3.65	5.56	**5.65**	**3.34**	2.42	-0.25	**1.25**	0.90	-0.74
$RW^{\sigma_t,\rho}_{ncp}$	**1.36**	0.21	0.69	**4.01**	**5.75**	2.78	3.13	2.20	-0.75	1.09	0.99	-0.64
$RW^{\sigma_t,\rho_t}_{ncp}$	1.19	0.25	0.82	3.88	5.70	2.92	3.18	2.27	-0.64	0.99	0.87	-0.56
$VECM^{r0,\sigma,\rho_t}$	-0.03	1.00	0.39	-1.53	-0.79	-0.49	-0.91	-0.60	**0.13**	-0.51	-0.26	**0.67**
$VECM^{r1,\sigma,\rho_t}_{ncp}$	-0.45	-0.43	-7.89	-0.49	-0.38	-4.69	-1.71	-1.77	-7.81	-1.14	-0.04	0.53
$VECM^{r2,\sigma,\rho_t}_{ncp}$	-0.01	**1.74**	-0.32	-1.12	-1.32	-2.27	-0.72	-1.87	-11.92	-0.59	-0.20	-1.00
$VECM^{r3,\sigma_t,\rho}$	0.07	1.55	-6.70	0.09	-0.41	-1.00	0.46	0.02	-17.96	0.63	-0.15	-0.89
$VECM^{r4,\sigma,\rho_t}$	0.43	-1.57	-9.05	-1.49	-1.81	-4.37	-1.13	-3.26	-25.93	-0.41	0.49	-3.99
ETS^{σ}	0.07	0.69	**4.92**	-2.18	-0.35	0.19	1.69	-0.70	-4.04	-0.06	-1.02	-2.79

Coloring w.r.t. test statistic: <-5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | >5

RMSE	EUA			NGas			Oil			Coal		
Model	H1	H5	H30	H1	H5	H30	H1	H5	H30	H1	H5	H30
$RW^{\sigma,\rho}$	0.03	0.07	0.14	**0.03**	0.07	0.18	0.05	**0.13**	0.31	0.03	0.07	0.22
$RW^{\sigma_t,\rho}_{lev,ncp}$	-0.17	0.02	0.07	-0.51	-0.07	0.02	-0.15	-0.32	**0.13**	0.05	0.02	0.12
$RW^{\sigma_t,\rho_t}_{lev,ncp}$	0.18	0.12	-0.01	-0.14	0.00	-0.13	-0.09	-0.53	-0.30	0.33	-0.12	0.02
$RW^{\sigma_t,\rho}_{ncp}$	0.27	0.03	0.15	-0.21	-0.07	0.21	-0.11	-0.10	-0.12	-0.05	0.30	0.12
$RW^{\sigma_t,\rho_t}_{ncp}$	-0.08	0.15	0.15	-0.26	-0.10	-0.15	-0.27	-0.15	-0.03	-0.35	0.15	-0.07
$VECM^{r0,\sigma,\rho_t}$	-0.26	0.69	0.65	-1.46	-0.56	-0.03	0.01	-1.03	-0.32	0.05	-0.40	0.03
$VECM^{r1,\sigma,\rho_t}_{ncp}$	-1.45	-0.66	-7.84	-0.78	-0.85	-3.81	-0.34	-1.65	-5.48	-0.64	-0.32	2.59
$VECM^{r2,\sigma,\rho_t}_{ncp}$	0.17	**1.25**	-0.66	-1.33	-2.03	-2.72	**0.82**	-1.95	-8.55	-0.17	0.16	1.34
$VECM^{r3,\sigma_t,\rho}$	-0.51	0.70	-8.22	-1.16	-1.59	-1.14	-0.18	-1.46	-13.24	0.01	-0.58	1.74
$VECM^{r4,\sigma,\rho_t}$	-0.41	0.00	-10.01	-1.54	-2.58	-3.39	0.63	-2.40	-16.85	-0.03	**0.88**	**2.90**
ETS^{σ}	**0.52**	0.90	**4.70**	-0.69	**0.65**	**0.48**	0.20	-1.15	-2.91	**0.60**	-4.18	-11.68

Coloring w.r.t. test statistic: <-5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 | >5

- The best performing model is $\text{RW}^{\rho_t,\sigma_t}_{\text{ncp,lev}}$,
- The temporal evolution of volatilities $\sigma^2_{i,t}$ gives information about the evolution of uncertainties in the market.
- The linear dependencies $\Lambda(\rho_{ij,t})$ give information about the connection between markets.


Sigma
0.50000
0.12500
0.03125
2016
2018
2020
2022
EUA
Oil
NGas
Coal

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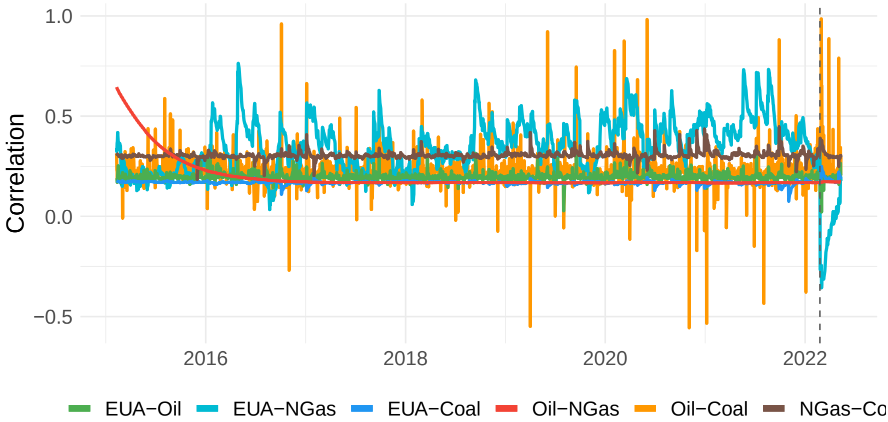

Discussion & Outlook

Conclusion

- For the log-prices cointegration is not important for forecasting,
- Leverage volatility modeling enhances performance,
- Time varying dependence parameters also improve multivariate forecast.
- Leverage and skewness are also important.

- Since the beginning of the russian invasion of Ukraine, volatility increases,
- The linear dependence between EUA – NGas drops during the beginning of the war,
- The dependence relaxes shortly afterwards.

Outlook

- Repeat analysis in levels,
- Use other transformations beforehand.
- Consider other copula models (e.g. Vine copulas)
- Use more/other data.

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Thanks for your Attention!