A survey of electricity spot and futures price models for risk management applications

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Introduction: how to choose the best model ?

- **Practical needs**
  - The model must reproduce the main characteristics of prices;
  - The model must be simple enough to be used in practice;
  - The model must have a calibration method.
- **Two sources of information**
  - The **Observed data** (their value, their volatility...);
  - The **Expertise** (practical knowledge of how the price is built).

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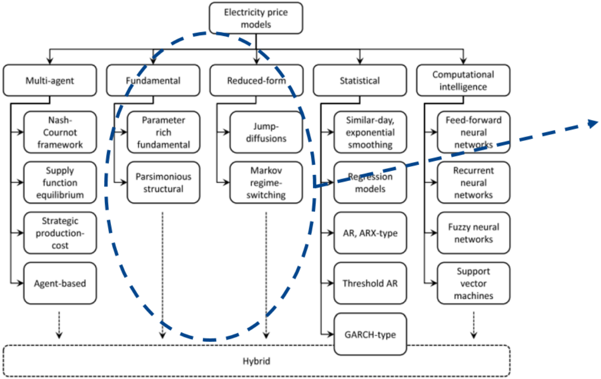
All models are wrong!

- The choice of (the class of) model is mainly driven by the targeted application:
  - Why do we need a model ?
  - Which constraints due to the model choice ?
- One particular context: finance
  - Implies specific constraints on the model,
  - Needs the modelling on futures prices as well as spot prices.

Taxonomy of modelling approaches and their applications



Applications:

- Risk management
- Pricing contracts / derivatives

Source : R. Weron, *Electricity price forecasting: A review of the state-of-the-art with a look into the future*, International Journal of Forecasting 30(4): 1030-1081, 2014.

Survey on electricity price models for risk management purposes

Objectives

- Give an (as) exhaustive (as possible) review on all the existing approaches to depict electricity prices for financial risk management purposes;
- Propose a classification based on practical questions;
- Unify the notation to ease the understanding and possible comparisons.

Proposed classification

Classification related to the expected behaviour of prices

- **Gaussian factor models** in which the uncertainty is modeled by Brownian motions;
- **Models to represent price spikes** which are by far the most numerous with a wide variety of approaches;
- **Stochastic volatility models** which are a classical extension of Gaussian factor models;
- **Multidimensional models** that seek to represent a set of prices in which one is the price of electricity;
- **Structural models** which can naturally be seen as multidimensional models, but in which the link between variables is represented by complex functions linked to market fundamentals.

Around 170 papers: 135 journal papers, 5 conference papers, 15 book chapters and 14 unpublished papers)

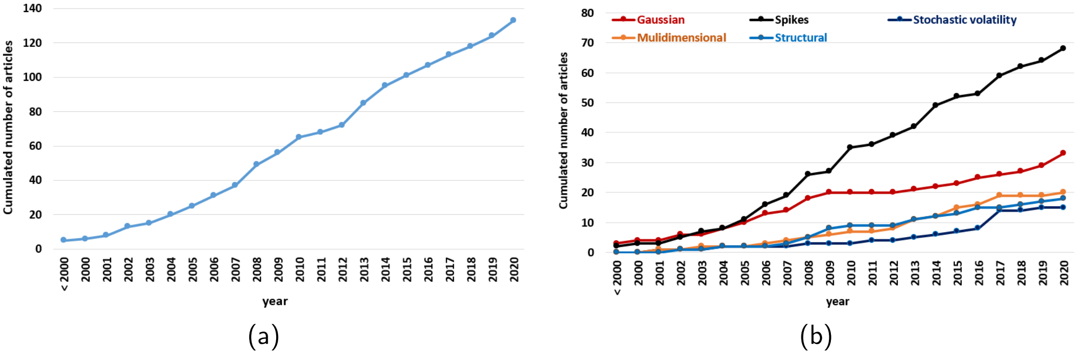


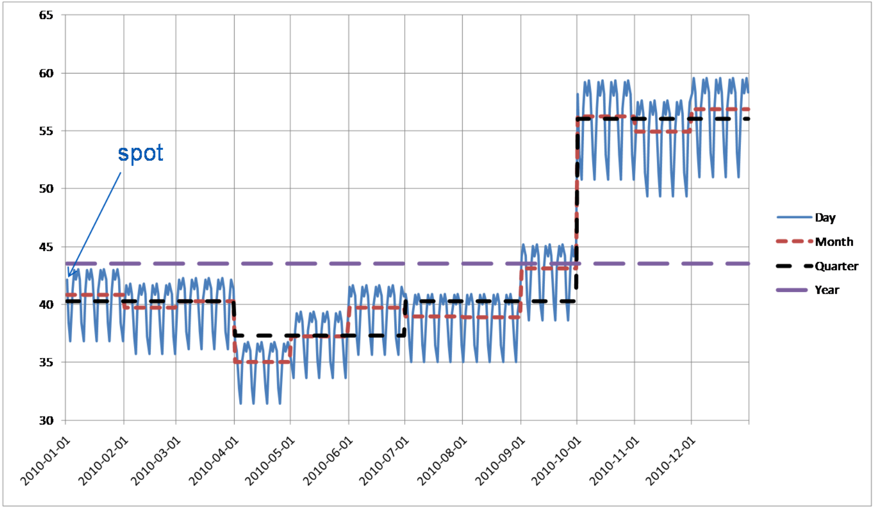
Figure: Evolution by year of the cumulative number of papers in electricity price modeling for risk management: (a) global view and (b) view by model family.

Key notation

Relationship between spot prices and forward products with delivery (FPDs)

- **Notation**
  - S_t : spot price quoted at date t
  - $F_t(T_1, T_2)$: forward contract (FPD) quoted at date t, delivering 1MWh of power for all hours between T_1 and T_2
- **Key link:** curve of unitary forward contract (Forward curve)
  - unitary forward price: $f_t(T) = F_t(T, T + 1h)$: forward price quoted at date t, for delivering 1MWh of power at maturity T.
  - forward curve $(f_t(T))_{T \geq t}$
- Observed contracts from unitary forward prices
  - Spot price as a limit: $S_t = \lim_{T \to t} f_t(T)$
  - Forward contracts by no-arbitrage: $F_t(T_1, T_2) = \int_{T_1}^{T_2} w(s) f_t(s) ds$

Example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2010-11-01
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Day
Month
Quarter
Year

Gaussian factor models

Schwartz-type models and the convenience yield approach

Model	1 factor	2 factors	3 factors
[Schwartz, 1997]	$d\ln S_t = \alpha(\beta - \ln S_t)dt + \sigma dW_t$	$\frac{dS_t}{S_t} = (\mu - \delta_t)dt + \sigma_1 dW_t^1$ $d\delta_t = \alpha(\beta - \delta_t)dt + \sigma_2 dW_t^2$	$\frac{dS_t}{S_t} = (r_t - \delta_t)dt + \sigma_1 dW_t^1$ $d\delta_t = \alpha(\beta - \delta_t)dt + \sigma_2 dW_t^2$ $dr_t = \kappa(\mu - r_t)dt + \sigma_3 dW_t^3$
[Gibson and Schwartz, 1990]	-	$d\ln S_t = (\mu - \delta_t)dt + \sigma_1 dW_t^1$ $d\delta_t = \alpha(\beta - \delta_t)dt + \sigma_2 dW_t^2$	-
[Schwartz and Smith, 2000]	-	$\ln S_t = X_t^1 + X_t^2$ $dX_t^1 = -\alpha_1 X_t^1 dt + \sigma_1 dW_t^1$ $dX_t^2 = \mu dt + \sigma_2 dW_t^2$	-
[Lucia and Schwartz, 2002]	$P_t = S_t$ or $P_t = \ln S_t$ $P_t = \Lambda(t) + X_t$ $dX_t = -\alpha X_t dt + \sigma dW_t$	$P_t = S_t$ or $P_t = \ln S_t$ $P_t = \Lambda(t) + X_t^1 + X_t^2$ $dX_t^1 = -\alpha X_t^1 dt + \sigma^1 dW_t$ $dX_t^2 = \mu dt + \sigma^2 dW_t$	-

Heath-Jarrow-Morton (HJM) models

- Principles of interest rates models [Gabillon, 1991, Benth and Koekebakker, 2008]:

$$\frac{df_t(T)}{f_t(T)} = \sum_{i=1}^{N} \sigma_n(t, T) dW_t^n \quad (1)$$

- Most commonly used in practice : multi-factor HJM with exponential volatility [Clewlow and Strickland, 1999, Manoliu and Tompaidis, 2002, Diko et al., 2006, Kiesel et al., 2009, Edoli et al., 2013, Kremer et al., 2020]

$$\frac{df_t(T)}{f_t(T)} = \sum_{i=1}^{N} \sigma_n e^{-\alpha_n(T-t)} dW_t^n \quad (2)$$

  - Able to represent the Samuelson effect,
  - Classic methods to calibrate the model [Kiesel et al., 2009, Edoli et al., 2013, Féron and Daboussi, 2015, Féron et al., 2019, Guerini et al., 2020, Fabbiani et al., 2021],
  - Lots of analytical formulas for derivatives (Call options...),
  - Recent work [Féron and Gruet, 2020] shows that the optimal number of factors to represent futures prices in the Western European countries is $N = 5$ (according to *Bayesian Information Criterion*).

Gaussian factor models

- Gaussian factor models are the simplest models
  - in terms of the (Gaussian) representation of the uncertainty,
  - in terms of getting simple and analytical formulas for risk management applications (option values, greeks...);
- They often offer the possibility to access to classic calibration procedures;
- There is a wide range of literature (both in energy markets and on interest rate models) studying this class of models;
- They are still the preferred models for practitioners;
- The main drawback is the difficulty, if not impossibility, to model spikes.

Modelling spikes

Spikes in electricity prices

- Johnson and Barz (99) state as early as 1999 the need to represent positive and negative jumps as fundamental characteristics of the prices observed in the electricity markets of California, Australia, and Scandinavia;
- More than 100 articles in the literature talk about the objective of modelling spikes;
- Price spikes are often related to the non-storability of electricity and events of very high tension in the system, followed by a very quick return to equilibrium;
- Modelling price spikes is very important for the pricing and financial management of "peak power plants" (Out-of-the-money Call option).

Most classical approach to depict electricity spikes

Same as in classical finance: replace Brownian motions by more general **Lévy processes**

Lévy processes for electricity spikes modelling

- **Simple idea:** replace Brownian motion W by Lévy process L:

$$
\begin{aligned}
P_t &= \Lambda(t) + \sum_{n=1}^{N} X_t^n, \\
dX_t^n &= -\alpha_n X_t^n dt + dL_t^n,
\end{aligned}
\tag{3}
$$

  - Lots of papers study this class of models (calibration procedures, futures and derivatives pricing, hedging complex contracts...) [Benth et al., 2007, Benth et al., 2008, Hess, 2014, Hess, 2017];
  - Needs of high level of Mathematics.

- Most popular classes of Lévy processes used:
  - Compound Poisson processes [Johnson and Barz, 1999, Deng, 2000, Cartea and Figueroa, 2005, Pawłowski and Nowak, 2015, Deschatre et al., 2018];
  - Normal Inverse Gaussian processes [Benth and Šaltytė Benth, 2004, Collet et al., 2006, Goutte et al., 2014, Benth and Schmeck, 2014].

Other way to depict spikes: regime switching models

[Deng, 2000, Kholodnyi, 2005, Bierbrauer et al., 2007, Kholodnyi, 2004, Kholodnyi, 2008, Janczura, 2014]

- **Main principle of regime switching models: use a (hidden) state variable that describes the "state" of the system**
  - A specific state is built to define the "spike" state: the value of the price must be high,
  - One or several states represent more normal conditions,
  - Some papers add a state for negative prices;
- Easy way to depict extreme events;
- Possibility to define the change in state depending on exogenous variables (weather, electricity consumption, renewable production...);
- but, still difficult to compute futures and derivatives prices.

Multidimensional models

Global idea of multidimensional models

- Assets are often exposed to several markets
  - (simplified) Financial vision of a power plant:

$$\left(S_t^{power} - hS_t^{fuel} - h'S_t^{C02}\right)^+, \quad \forall t \in [0\ ;\ T] \tag{4}$$

  - Interconnection line between two markets A and B

$$\left(S_t^{A} - S_t^{B}\right)^+ + \left(S_t^{B} - S_t^{A}\right)^+, \quad \forall t \in [0\ ;\ T] \tag{5}$$

- => need to define a joint model for all the risks of the asset

Multidimensional models

- **Correlation model:** just duplicate the one-dimension model for each risk and consider (estimate) a correlation coefficient between the Brownian motions [Deng et al., 2001]

$$\frac{dF_t^d(T)}{F_t^d(T)} = \mu^d dt + \sigma^d dW_t^d, \quad d = 1, 2,$$

- **Coupling factor models:** use a common factor in the dynamics to depict a common behaviour [Frikha and Lemaire, 2013]

$$S_t^1 = \Lambda^1(t) + X_t^1 + Z_t,$$
$$S_t^2 = \Lambda^2(t) + X_t^2 + Z_t,$$

Multidimensional models

- **Cointegration models:** stationary representation of a linear combination of two processes, although the individual depiction of each process is non-stationary
  - Nakajima and Ohashi [Nakajima and Ohashi, 2012] generalized the Gibson and Schwartz [Gibson and Schwartz, 1990] model with cointegration:

$$d\log(S_t^d) = \left(r - \frac{\sigma_{S^d}^2}{2} - \delta_t^d + b_d z_t\right) dt + \sigma_{S^d} dW_t^{S^d},$$
$$d\delta_t^d = \alpha_d \left(\mu_d - \delta_t^d\right) dt + \sigma_{\delta^d} dW_t^{\delta^d},$$

for $d = 1, \ldots, D$, all the Brownian motions were correlated and z_t a (stationary) error correction term related to cointegration, designed as a linear combination of the $\log(S_t^d)$.

Structural models

Global principle of structural models

- Proposed definition (and properties) of structural model
  - the spot price is written as a function of fundamental variables;
  - this function aims at reproducing the way the spot price is calculated, that is, by reproducing the principles of a balance between supply and demand;
  - and the variables used are observable and their model is calibrated with observed data.

- Structural models can be embedded into other classes:
  - Structural models are using Gaussian factors to represent the fundamental variables. The (complex) function to build spot prices allows to depict spikes,
  - Structural models are, by construction, multidimensional models;

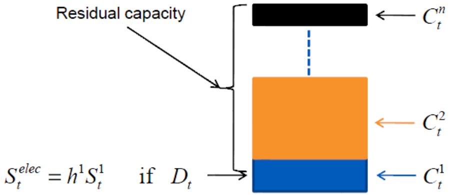
- In appendix: table summarizing all the structural models developed in the literature;

- Most important work: [Carmona et al., 2013, Aïd et al., 2009, Aïd et al., 2013] .

Structural model: price as the marginal cost [Aïd et al., 2009]

- **Variables**
  - n fuels, $1 \leq i \leq n$
  - D_t demand (in MW)
  - C_t^i production capacities (in MW)
  - S_t^i fuel prices (in €/MWh)
  - h^i heat rates ($h^i S_t^i$ in €/MWh, increasing with i)

- **Principle**



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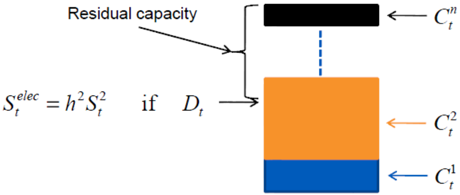

Structural model: price as the marginal cost [Aïd et al., 2009]

- Summarizing:

$$S_t = \sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{D_t \in I_t^i}$$

with $I_t^i = \left] \sum_{k=1}^{i-1} C_t^k \; ; \; \sum_{k=1}^{i} C_t^k \right]$

- Computing forward prices:

$$f_t(T) = \sum_{i=1}^{n} \mathbb{P}^* \left[D_T \in I_T^i | \mathcal{F}_t \right] h_i f_t^i(T)$$

- Simple interpretation: the power future price is the weighted averaged of futures fuel, the weights corresponding to the percentage of marginality for each technology

Summary

Conclusion and a view toward future work

Conclusion

- Gaussian factor models are the preferred approach for risk management applications;
- The use of Lévy processes has allowed many researchers to enrich the dynamics of spot prices to represent these spikes, but at the cost of increasing the complexity of the derivatives calculation;
- The consideration of multidimensional models is very useful in practice, given the natural exposure of energy assets to several markets:
  - The dependency structure between the different prices modeled is very important. The calibration may be difficult,
  - Approaches in the literature are very classical: simple correlation, coupling factors, cointegration;
- Structural models seem promising for the representation of spot prices but make risk management complex:
  - They have benefited from thorough treatments, for example by Aïd et al. [Aïd et al., 2013],
  - The calibration of structural models is also complex, since all the uncertainties used for the fundamental representation must be the subject of a modeling and estimation effort.

Future work

- A need of joint models on weather variables and power prices:
  - Expected increase of renewable energy => expected increase of weather impacts on power production => expected increase of power price volatility,
  - Expected increase of the impact of weather conditions on conventional thermal power plants (due to more frequent extreme climatic events),
  - Only a few papers attempting to depict the dependency between weather variables and power prices [Veraart, 2016, Deschatre and Veraart, 2017, Rowińska et al., 2020];
- A need to model interconnections:
  - Interconnections between markets expand, models develop to reproduce previously ignored facts, such as the equality of prices in multiple countries over multiple time periods,
  - The recent work of Gardini et al. [Gardini et al., 2020b, Gardini et al., 2020a] and Christensen and Benth [Christensen and Benth, 2020] are notable advances in incorporating the connected nature of markets into modeling, without using the complexity of structural models like in [Alasseur and Féron, 2018].

Future work

- A need to depict liquidity / market impact / market incompleteness:
  - Lack of liquidity that must be taken into account on trading strategies and hedging strategies,
  - Only a few works exist [Hess, 2017, Kremer et al., 2020];
- A need to model intraday prices:
  - The development of renewable production increases price risk in the intraday level,
  - The valuation of flexibilities must be computed on intraday prices,
  - The multi-horizon modeling of Hinderks et al. [Hinderks et al., 2020] demonstrates the opportunities and challenges in terms of modeling such prices together.

Structural models: summary

Structural models: summary (1/2)

Model	Fundamental variables considered	Futures prices computation	Markets studied
[Eydeland and Geman, 1999]	Demand Marginal fuel	Analytical	US markets
[Kosecki, 1999]	Demand Marginal fuel	Solution of PDE	US markets
[Barlow, 2002]	Demand	Semi analytical	Alberta California
[Burger et al., 2004]	Demand Maximal capacity 2 unobservable factors	Analytical with approximation	EEX
[Kanamura and Ohashi, 2007]	Demand	Semi analytical	PJM
[Pirrong and Jermakyan, 2008] [Pirrong, 2011]	Demand Marginal fuel (Outage)	Solution of PDE	PJM
[Coulon and Howison, 2009]	Demand Maximal capacity	Analytical case 1 fuel	NEPOOL PJM

Structural models: summary (2/2)

Model	Fundamental variables considered	Futures prices computation	Markets studied
[Aïd et al., 2009] [Aïd et al., 2013]	Demand K fuel costs K capacities	Analytical	France
[Lyle and Elliott, 2009]	Demand Baseload capacity	Analytical	Alberta
[de Maere d'Aertrycke and Smeers, 2010]	Demand Marginal fuel	Solution of PDE	Germany
[Coulon et al., 2013]	Demand Gas price 1 unobservable factor 1 state variable	Analytical	ERCOT
[Carmona et al., 2013]	Demand K fuel costs	Analytical	-
[Wagner, 2014]	Demand Wind production Solar production	Monte Carlo	Germany

References I

Aïd, R., Campi, L., and Langrené, N. (2013).
A structural risk-neutral model for pricing and hedging power derivatives.
Mathematical Finance, 23(3):387–438.

Aïd, R., Campi, L., Nguyen Huu, A., and Touzi, N. (2009).
A structural risk-neutral model of electricity prices.
International Journal of Theoretical and Applied Finance, 12(07):925–947.

Alasseur, C. and Féron, O. (2018).
Structural price model for coupled electricity markets.
Energy Economics, 75:104–119.

Barlow, M. T. (2002).
A diffusion model for electricity prices.
Mathematical Finance, 12(4):287–298.

Benth, F. E., Cartea, Á., and Kiesel, R. (2008).
Pricing forward contracts in power markets by the certainty equivalence principle: explaining the sign of the market risk premium.
Journal of Banking & Finance, 32(10):2006–2021.

Benth, F. E., Kallsen, J., and Meyer-Brandis, T. (2007).
A non-Gaussian Ornstein–Uhlenbeck process for electricity spot price modeling and derivatives pricing.
Applied Mathematical Finance, 14(2):153–169.

References II

Benth, F. E. and Koekebakker, S. (2008).
Stochastic modeling of financial electricity contracts.
Energy Economics, 30(3):1116–1157.

Benth, F. E. and Schmeck, M. D. (2014).
Pricing futures and options in electricity markets.
In *The Interrelationship Between Financial and Energy Markets*, pages 233–260. Springer.

Benth, F. E. and Šaltytė Benth, J. (2004).
The normal inverse Gaussian distribution and spot price modeling in energy markets.
International Journal of Theoretical and Applied Finance, 07(02):177–192.

Bierbrauer, M., Menn, C., Rachev, S. T., and Trück, S. (2007).
Spot and derivative pricing in the EEX power market.
Journal of Banking & Finance, 31(11):3462–3485.

Burger, M., Klar, B., Müller, A., and Schindlmayr, G. (2004).
A spot market model for pricing derivatives in electricity markets.
Quantitative Finance, 4(1):109–122.

Carmona, R., Coulon, M., and Schwarz, D. (2013).
Electricity price modeling and asset valuation: a multi-fuel structural approach.
Mathematics and Financial Economics, 7(2):167–202.

References III

Cartea, Á. and Figueroa, M. G. (2005).
Pricing in electricity markets: A mean reverting jump diffusion model with seasonality.
Applied Mathematical Finance, 12(4):313–335.

Christensen, T. S. and Benth, F. E. (2020).
Modelling the joint behaviour of electricity prices in interconnected markets.
Quantitative Finance, pages 1–16.

Clewlow, L. and Strickland, C. (1999).
Valuing energy options in a one factor model fitted to forward prices.
Research Paper Series 10, Quantitative Finance Research Centre, University of Technology, Sydney.

Collet, J., Duwig, V., and Oudjane, N. (2006).
Some non-Gaussian models for electricity spot prices.
In *2006 International Conference on Probabilistic Methods Applied to Power Systems*, pages 1–7. IEEE.

Coulon, M. and Howison, S. (2009).
Stochastic behavior of the electricity bid stack: from fundamental drivers to power prices.
The Journal of Energy Markets, 2(1):29–69.

Coulon, M., Powell, W. B., and Sircar, R. (2013).
A model for hedging load and price risk in the Texas electricity market.
Energy Economics, 40(C):976–988.

References IV

de Maere d'Aertrycke, G. and Smeers, Y. (2010).
The valuation of power futures based on optimal dispatch.
The Journal of Energy Markets, 3(3):27–50.

Deng, S. (2000).
Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes.
University of California Energy Institute Berkeley.

Deng, S., Johnson, B., and Sogomonian, A. (2001).
Exotic electricity options and the valuation of electricity generation and transmission assets.
Decision support systems, 30(3):383–392.

Deschatre, T., Féron, O., and Hoffmann, M. (2018).
Estimating fast mean-reverting jumps in electricity market models.
ESAIM: Probability and Statistics, 24.

Deschatre, T. and Veraart, A. E. D. (2017).
A joint model for electricity spot prices and wind penetration with dependence in the extremes.
In *Forecasting and Risk Management for Renewable Energy*, pages 185–207. Springer.

Diko, P., Lawford, S., and Limpens, V. (2006).
Risk premia in electricity forward prices.
Studies in Nonlinear Dynamics & Econometrics, 10(3).

References V

Edoli, E., Tasinato, D., and Vargiolu, T. (2013).
Calibration of a multifactor model for the forward markets of several commodities.
Optimization, 62(11):1553–1574.

Eydeland, A. and Geman, H. (1999).
Fundamendals of electricity derivatives.
In *Energy modelling and the management of uncertainty*, pages 35–43. Risk.

Fabbiani, E., Marziali, A., and De Nicolao, G. (2021).
Fast calibration of two-factor models for energy option pricing.
Applied Stochastic Models in Business and Industry, 1(11).

Féron, O. and Daboussi, E. (2015).
Calibration of electricity price models.
In *Commodities, Energy and Environmental Finance*, pages 183–210. Springer.

Féron, O. and Gruet, P. (2020).
Estimation of the number of factors in electricity price models.
Technical report, FiME Lab.

Féron, O., Gruet, P., and Hoffmann, M. (2019).
Efficient volatility estimation in a two-factor model.
Scandinavian Journal of Statistics, 47(3):862–898.

References VI

Frikha, N. and Lemaire, V. (2013).
Joint modelling of gas and electricity spot prices.
Applied Mathematical Finance, 20(1):69–93.

Gabillon, J. (1991).
The term structures of oil futures prices.
Oxford Institute for Energy Studies, WPM 17(286084).

Gardini, M., Sabino, P., and Sasso, E. (2020a).
A bivariate normal inverse Gaussian process with stochastic delay: efficient simulations and applications to energy markets.
arXiv preprint.

Gardini, M., Sabino, P., and Sasso, E. (2020b).
Correlating Lévy processes with self-decomposability: Applications to energy markets.
arXiv preprint.

Gibson, R. and Schwartz, E. S. (1990).
Stochastic convenience yield and the pricing of oil contingent claims.
The Journal of Finance, 45(3):959–976.

Goutte, S., Oudjane, N., and Russo, F. (2014).
Variance optimal hedging for continuous time additive processes and applications.
Stochastics An International Journal of Probability and Stochastic Processes, 86(1):147–185.

References VII

Guerini, A., Marziali, A., and De Nicolao, G. (2020).
MCMC calibration of spot-prices models in electricity markets.
Applied Stochastic Models in Business and Industry, 36(1):62–76.

Hess, M. (2014).
Pricing electricity derivatives under future information.
Available at SSRN.

Hess, M. (2017).
Optimal liquidation of electricity futures portfolios: An anticipative market impact model.
Available at SSRN.

Hinderks, W. J., Korn, R., and Wagner, A. (2020).
A structural Heath–Jarrow–Morton framework for consistent intraday spot and futures electricity prices.
Quantitative Finance, 20(3):347–357.

Janczura, J. (2014).
Pricing electricity derivatives within a Markov regime-switching model: a risk premium approach.
Mathematical Methods of Operations Research, 79(1):1–30.

Johnson, B. and Barz, G. (1999).
Selecting stochastic processes for modelling electricity prices.
In *Energy Modelling and The Management of Uncertainty*, pages 3–21. Jamson.

References VIII

Kanamura, T. and Ohashi, K. (2007).
A structural model for electricity prices with spikes: Measurement of spike risk and optimal policies for hydropower plant operation.
Energy Economics, 29(5):1010–1032.

Kholodnyi, V. A. (2004).
Valuation and hedging of European contingent claims on power with spikes: a non-Markovian approach.
Journal of Engineering Mathematics, 49(3):233–252.

Kholodnyi, V. A. (2005).
Modeling power forward prices for power with spikes: a non-Markovian approach.
Nonlinear Analysis: Theory, Methods & Applications, 63(5-7):958–965.

Kholodnyi, V. A. (2008).
The non-Markovian approach to the valuation and hedging of European contingent claims on power with scaling spikes.
Nonlinear Analysis: Hybrid Systems, 2(2):285–309.

Kiesel, R., Schindlmayr, G., and Börger, R. H. (2009).
A two-factor model for the electricity forward market.
Quantitative Finance, 9(3):279–287.

Kosecki, R. (1999).
Fuel-based power price modelling.
In *Energy Modelling and The Management of Uncertainty*, pages 91–98. Risk.

References IX

Kremer, M., Benth, F. E., Felten, B., and Kiesel, R. (2020).
Volatility and liquidity on high-frequency electricity futures markets: Empirical analysis and stochastic modeling.
International Journal of Theoretical and Applied Finance, 23.

Lyle, M. R. and Elliott, R. J. (2009).
A 'simple' hybrid model for power derivatives.
Energy Economics, 31(5):757–767.

Manoliu, M. and Tompaidis, S. (2002).
Energy futures prices: term structure models with Kalman filter estimation.
Applied Mathematical Finance, 9(1):21–43.

Nakajima, K. and Ohashi, K. (2012).
A cointegrated commodity pricing model.
Journal of Futures markets, 32(11):995–1033.

Pawłowski, M. and Nowak, P. (2015).
Modelling spot prices on the Polish energy market.
In Filev, D., Jabłkowski, J., Kacprzyk, J., Krawczak, M., Popchev, I., Rutkowski, L., Sgurev, V., Sotirova, E., Szynkarczyk, P., and Zadrozny, S., editors, *Intelligent Systems'2014*, pages 781–792. Springer International Publishing.

Pirrong, C. (2011).
Commodity Price Dynamics: A Structural Approach.
Cambridge University Press.

References X

Pirrong, C. and Jermakyan, M. (2008).
The price of power – the valuation of power and weather derivatives.
Journal of Banking & Finance, 32(12):2520–2529.

Rowińska, P. A., Veraart, A. E. D., and Gruet, P. (2020).
A multifactor approach to modelling the impact of wind energy on electricity spot prices.
submitted to Energy Economics.

Veraart, A. E. D. (2016).
Modelling the impact of wind power production on electricity prices by regime-switching Lévy semistationary processes.
In *Stochastics of Environmental and Financial Economics*, pages 321–340. Springer, Cham.

Wagner, A. (2014).
Residual demand modeling and application to electricity pricing.
The Energy Journal, 35(2).